Vehicle Dynamic Control Allocation for Path Following

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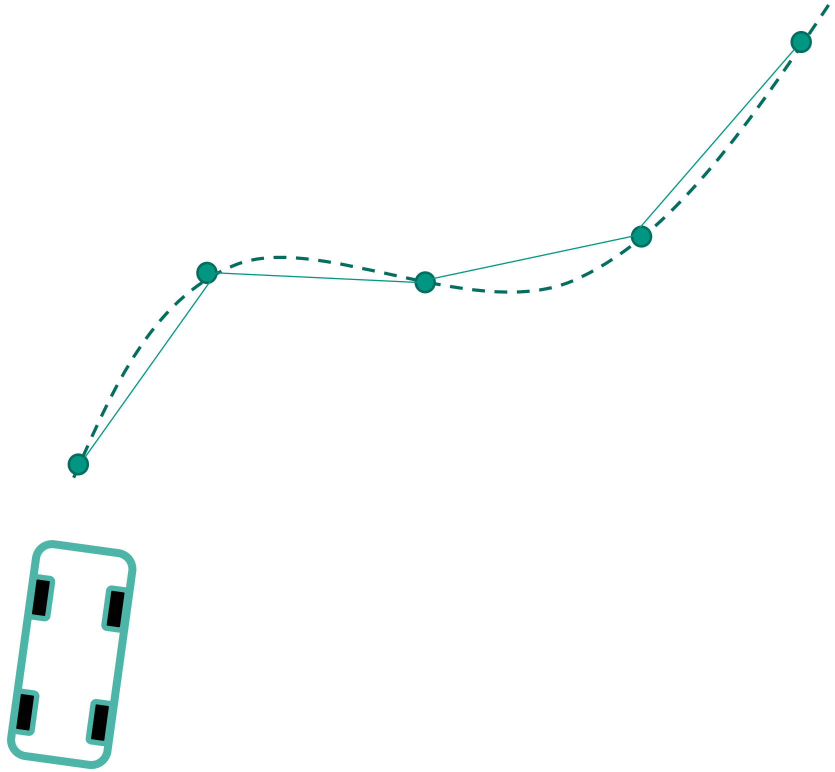
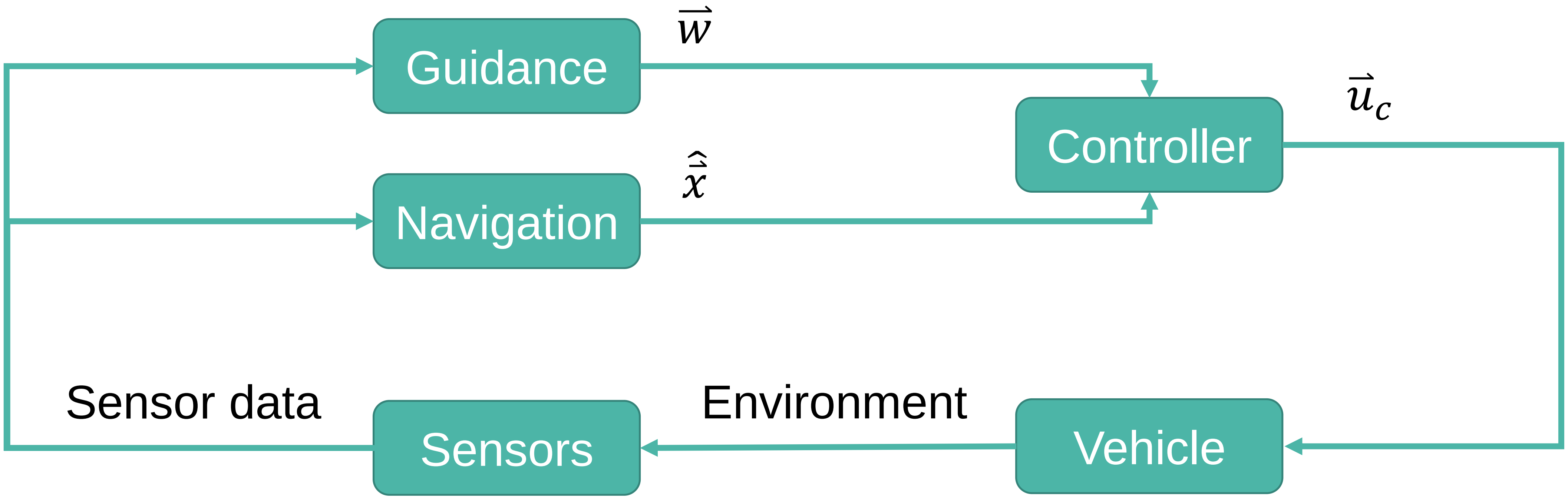
Motivation

- **System Design** for autonomous platforms
  - **Automotive Application:** System for path following
- **Path planning:** Straight lines, curve...
- **Guidance** problem
- **Controller** Structure



Introduction

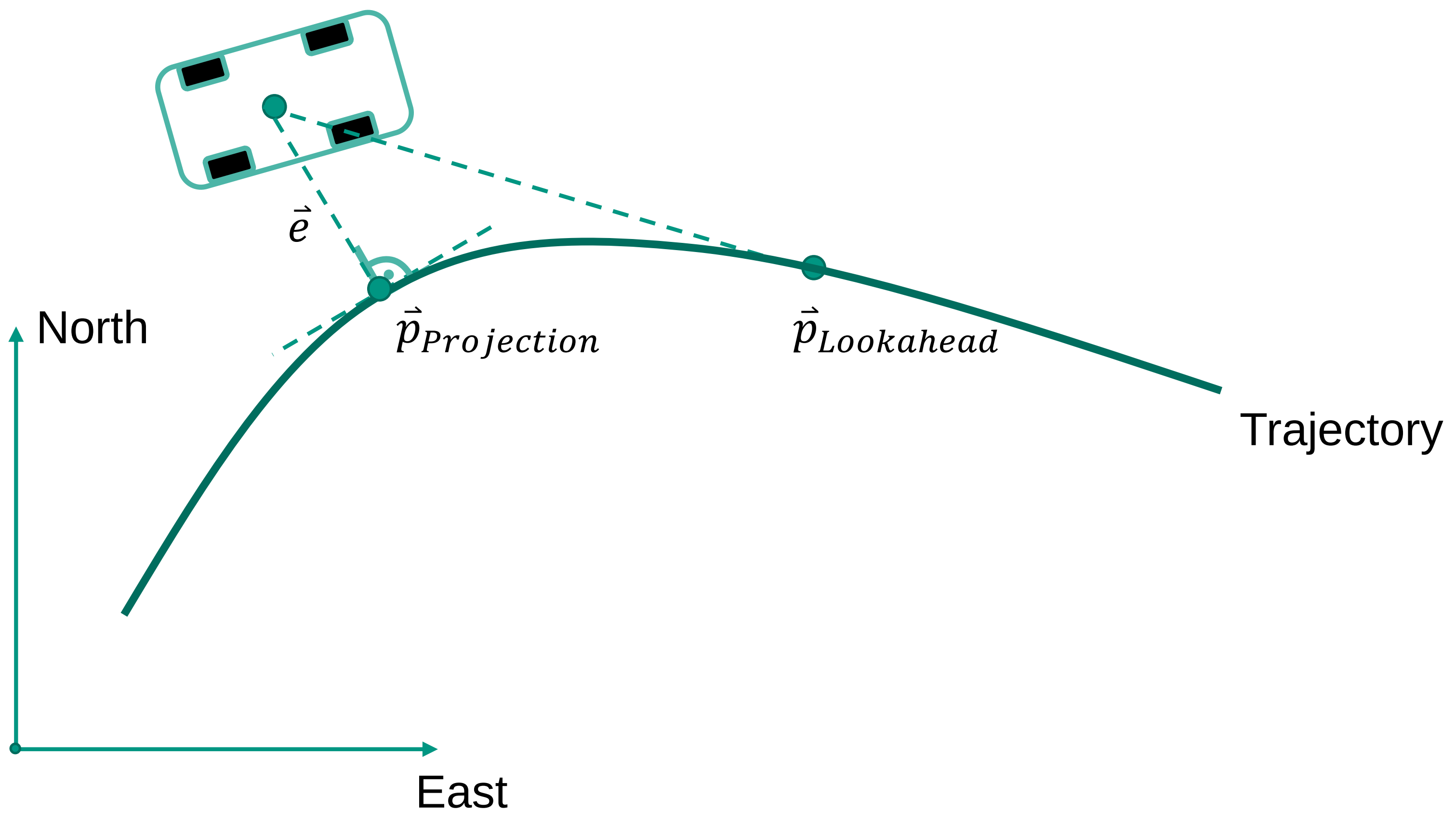


$\vec{w}$: Target values
$\hat{\vec{x}}$: Estimated actual values
$\vec{u}_c$: Actuator values

Outline

- Motivation and Introduction
- Guidance
- Controller
- Simulation Results
- Conclusion

Spline-Based Guidance



Input to guidance:

- Path (Spline data)
- Vehicle state
  - x_{Nav}
  - y_{Nav}

Objective Path Following:

- Minimize cross-track error (lateral deviation)

Spline-Based Guidance

1. Find **closest point** on trajectory (Newton-Raphson method)

$$y_n - y_d(\Theta) = -\frac{1}{\frac{\dot{y}_d(\Theta)}{\dot{x}_d(\Theta)}}\left(x_n - x_d(\Theta)\right)$$

$$\dot{y}_d(\Theta^*)\left(y - y_d(\Theta^*)\right) + \dot{x}_d(\Theta^*)\left(x - x_d(\Theta^*)\right) = 0$$

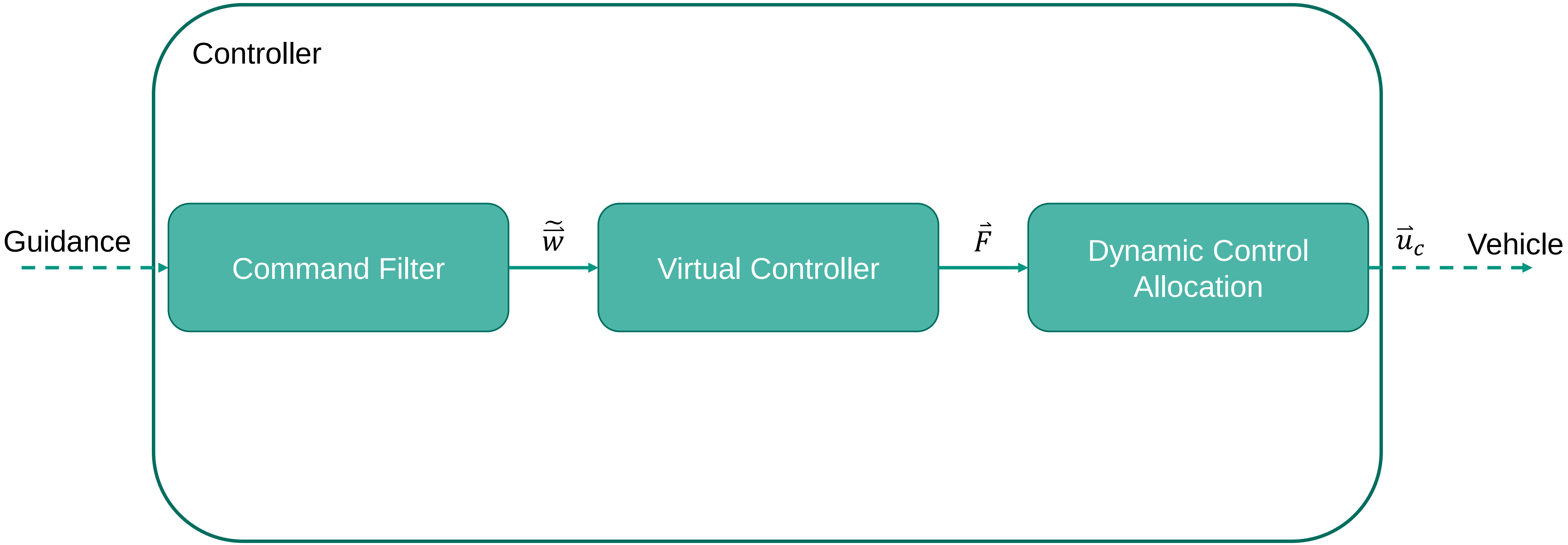
2. Move forward on the spline by increasing the **parametrizing variable $\boldsymbol{\Theta}^*$**

$$y_d(\Theta^* + \Delta) = a \cdot (\Theta^* + \Delta)^3 + b \cdot (\Theta^* + \Delta)^2 + c \cdot (\Theta^* + \Delta) + d$$

3. Calculate **desired course angle $\boldsymbol{\chi_d}$**

Δ: Look−ahead distance
Θ: Parametrizing variable

Controller Structure



Command Filter

- Receives commanded values from guidance
- Smooths and limits the values by taking vehicle dynamics into account
- Supplies the inputs to the virtual controller:
  - Commanded position $\vec{r}_c$
  - Commanded velocity $\vec{v}_c$
  - Commanded acceleration $\vec{a}_c$

$$\ddot{y} = \left[2D\omega_0 \left[\left[\frac{\omega_o}{2D}(e-y)\right] - \dot{y}\right]\right]$$

D: Damping coefficient
ω_0: Circular frequency

➡ Output signals are continuous and reasonable w.r.t. vehicle dynamics

Virtual Controller

- ”Virtual” with the meaning of not actually controlling the vehicle
- Calculates the necessary force in body-frame
- Derivation of the control law:

Translation dynamic:

$$\begin{bmatrix} \ddot{x}^n \\ \ddot{y}^n \end{bmatrix} = \frac{1}{m} \begin{bmatrix} F^n_{v,x} + F^n_{h,x} - F^n_{W,x} \\ F^n_{v,y} + F^n_{h,y} - F^n_{W,y} \end{bmatrix}$$

$$\dot{\vec{v}}^n_{eb} = \frac{1}{m} \left[C^n_b \left(C^b_s \vec{F}^s_v + \vec{F}^b_h \right) - \vec{F}^n_W \right]$$

$F^n_{v,x/y}$: Force front axle
$F^n_{h,x/y}$: Force back axle
$F^n_{W,x/y}$: Air resistance

Virtual Controller

- Define error dynamics

$$\dot{\vec{e}}_p = \vec{e}_v$$

$$\dot{\vec{e}}_v = \ddot{\vec{p}}_c - \dot{\vec{v}} = \ddot{\vec{p}}_c - \frac{1}{m}\left(C_b^n \vec{F}^b - \vec{F}_W^n\right)$$

- Apply backstepping control with Lyapunow

- Control law:

$$F_{Virtual} = C_n^b \cdot [m \cdot (\ddot{r}_{desired} + (\mathrm{K}_1 + \mathrm{K}_2) \cdot \tilde{v} + (I_{2x2} + K_1 \cdot K_2) \cdot \tilde{r}) + F_W]$$

K_i: Controller parameter

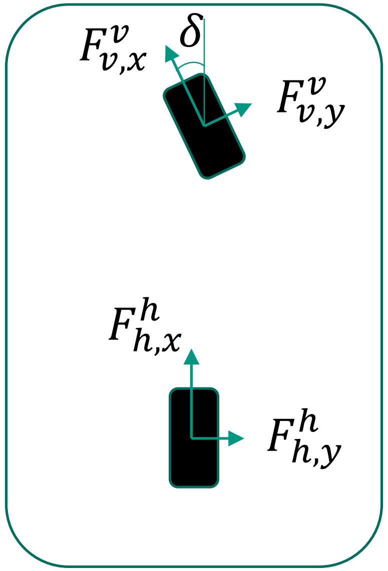
Dynamic Control Allocation

- **Control Allocation (CA):** Distribute forces/moments among longitudinal and lateral forces of individual tires.
- Distribution in an optimal manner (Cost function)
- Two CA techniques
  - **Static** Control Allocation
  - **Dynamic** Control Allocation

➡ Optimization problem with eq. constraint:

$$\min_{\vec{u}} J(\vec{u}, \vec{x}, t)$$

$$\vec{\tau}_d = \vec{h}(\vec{u})$$



Dynamic Control Allocation

- Objective function:

$$J(\vec{x},\vec{u},t) = \vec{u}^T W \vec{u} - w_F \sum_{i=1}^{4} \{\ln[-c_i(F_i)]\} - w_\delta(\ln[-c_5(\delta)] + \ln[-c_6(\delta)])$$

- 1. Part: Minimizing actuators $\vec{u} = \left[F_{v,x}^v, F_{v,y}^v, F_{h,x}^h, F_{h,y}^h, \delta\right]$
- 2. Part: **Barrier functions** penalizing the objective function when exceeding certain force and steering angle areas
- Extend OP with Lagrange multipliers to Lagrange function:

$$L\left(\vec{x},\vec{u},\vec{\lambda}t\right) = J(\vec{x},\vec{u},t) + \vec{\lambda}^T\left[\vec{\tau}_d(\vec{x},t) - \vec{h}(\vec{u})\right]$$

Dynamic Control Allocation

- Dynamic update law:

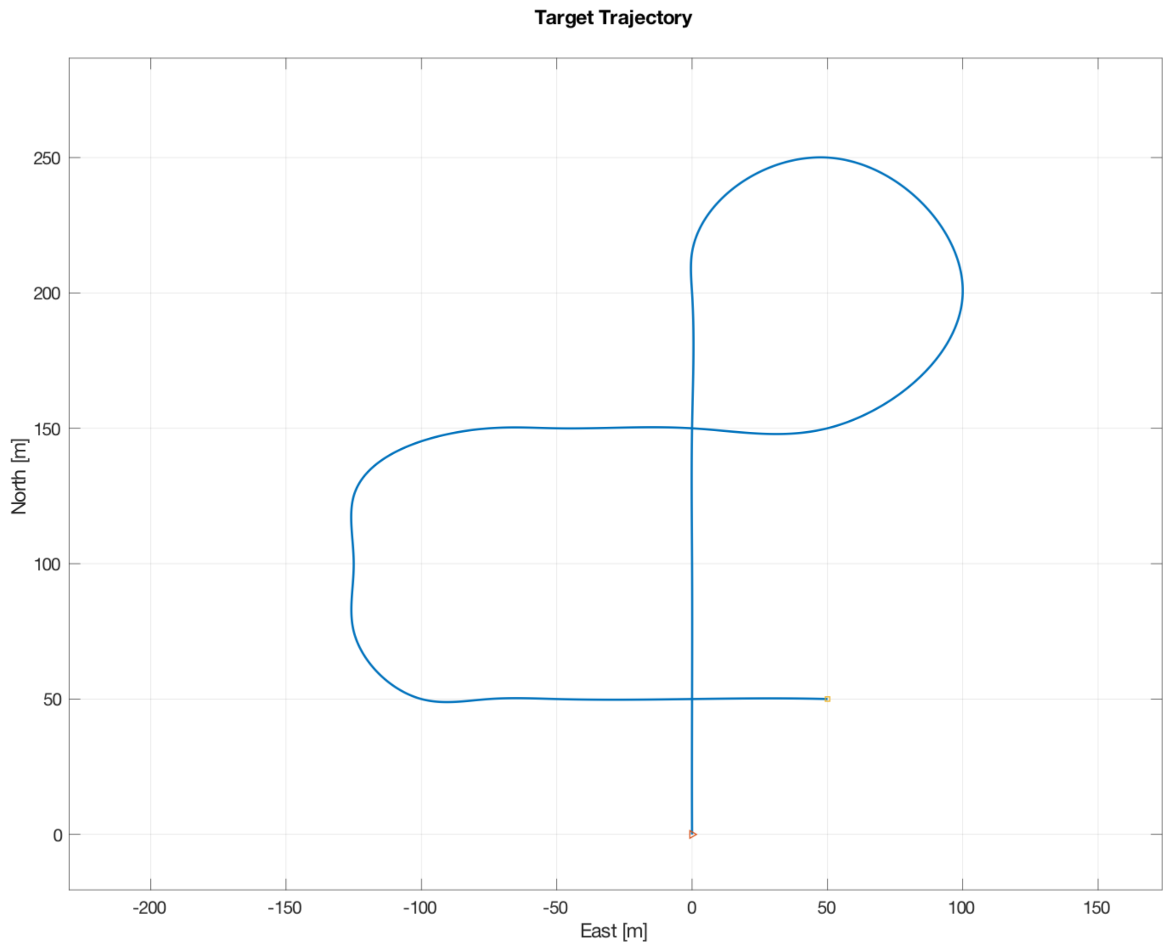
$$\begin{bmatrix} \dot{\vec{u}} \\ \dot{\vec{\lambda}} \end{bmatrix} = -\Gamma \mathrm{H} \begin{bmatrix} \frac{\partial L}{\partial \vec{u}} \\ \frac{\partial L}{\partial \vec{\lambda}} \end{bmatrix} + \mathrm{H}^{-1} \begin{bmatrix} \frac{\partial^2 L}{\partial \vec{\tau}_d \partial \vec{u}} \\ \frac{\partial^2 L}{\partial \vec{\tau}_d \partial \vec{\lambda}} \end{bmatrix} \dot{\vec{\tau}}_d$$

$$\Gamma = \gamma (\mathrm{H} W \mathrm{H} + \varepsilon I)^{-1} \qquad \mathrm{H} = \begin{bmatrix} \frac{\partial^2 L}{\partial \vec{u}^2} & \frac{\partial^2 L}{\partial \vec{u} \partial \vec{\lambda}} \\ \frac{\partial^2 L}{\partial \vec{\lambda} \partial \vec{u}} & \vec{0} \end{bmatrix}$$

Simulation Results

- Simulation using **CarMaker**
- **Monte-Carlo simulations** varying sensor noise to prove robustness of the system

- **Target Trajectory**
  - 200m Straight Line
  - 270° Curve with R=50m
  - 125m Straight Line
  - 90° Curve with R=25m
  - 50m Straight Line
  - 90° Curve with R=25m
  - 125m Straight Line



Conclusion

- Design of a system for path following in automotive applications using
  - Spline-based guidance with time-varying lookahead distance,
  - Backstepping controller,
  - And dynamic control allocation.

- Future goals
  - Improve robustness of the guidance
  - Introduce speed control to the guidance
  - Reduce and simplify amount of tuneable parameters