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Trajectory Planning for Automated Lane Changes

Assistance systems in modern cars are playing an increasingly important role. In addition to making the driver's job easier, they provide increased safety in traffic. At the TU Darmstadt, a system for automated lane change was developed with combined longitudinal and lateral control on the highway in cooperation with the Robert Bosch GmbH.

1 MOTIVATION

The most relevant criterion when comparing systems for automated lane changes is the number of covered scenarios. Simple automated lane changes were carried out as early as 1994 in the Project VITA II [1]. The lane changes in these early projects were primarily based on predefined trajectories, which were only slightly adapted, for example to the lane width. Provided that a sufficiently large area beside the own vehicle was free, the lane change could be carried out.

In recent works the trajectory is more adapted to the surrounding situation. Especially the longitudinal trajectory is varied. Combining longitudinal and lateral planning the longitudinal and lateral trajectory can be better geared to each other. Depending on the chosen longitudinal trajectory a different time is available for the lateral movement. Therefore, it is advantageous to plan a trajectory ($x(t)$, $y(t)$) instead of a path $y(x)$. Often the trajectory results from the solution of an optimisation problem, whose solution cannot be calculated analytically but is determined by an iterative solver. Because of the cyclic replanning of the trajectory this corresponds to a model predictive control.

Different approaches exist for the representation of the trajectory and its parameters. One possibility is using a polynomial of 5th or 7th degree for longitudinal and lateral direction. The polynomial is hereby defined by its boundary conditions. To generate different trajectories various discrete values for end position and end time are used. All possible combinations are evaluated and the one with the best objective value is chosen. This approach is used in the publications [2, 3]. To plan a lane change trajectory it is assumed here that the lateral motion can begin at the starting point.

If the computation time is less important and the focus is on flexibility, it is preferable to use a trajectory representation based on a fixed amount of temporally equidistant sampling points. The values at these sampling points are determined by the help of numerical solvers in such a way that the objective function reaches its minimum [4, 5]. This approach is very flexible, but needs relatively long computation time.

The aim of this project from TU Darmstadt and Robert Bosch GmbH is the development of a system for automated lane changes on motorways. The vehicle has to be guided in lateral as well as in longitudinal direction. Thus, in the future this system can also be used for merging onto the motorway. The assistance system has to guide the vehicle only during the lane change. After the process has been finished, the vehicle will be handed over to the automated longitudinal and lateral guidance system. To implement the algorithm on conventional electronic control units, strong focus has been set on small computation time. It will be assumed that the vehicle is equipped with four Mid Range Radar (MRR) sensors, one Long Range Radar (LRR) sensor to detect the surrounding vehicles and a camera to detect the lane markings, **FIGURE 1**.

The method described in this paper allows planning a trajectory whose lateral motion begins at a future point in time. At the same time the degrees of freedom of the trajectory are limited in such a way that it is possible to perform the calculations on an electronic control unit. The desired state at the end of the lane change manoeuvre is not chosen from a discrete set of points, instead it is determined by a solver. Because it is a comfort function only low dynamics are necessary. Therefore, the vehicle dynamics are modelled by a point mass to reduce complexity.

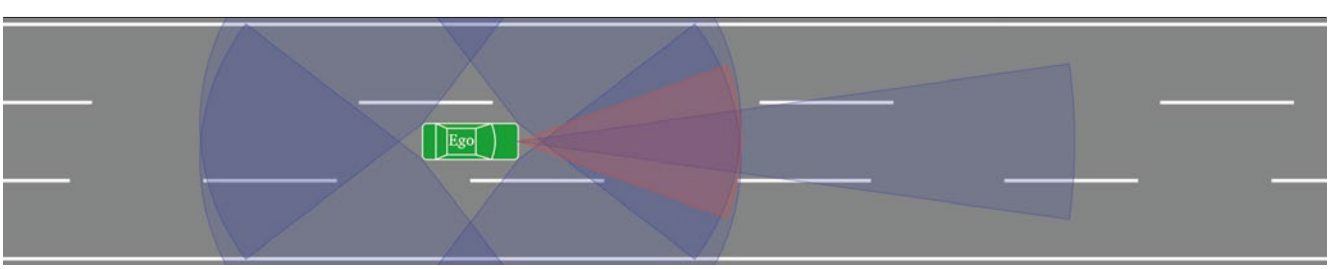

FIGURE 1 Vehicle sensors: radar blue, video red (ranges not to scale) (© TU Darmstadt)

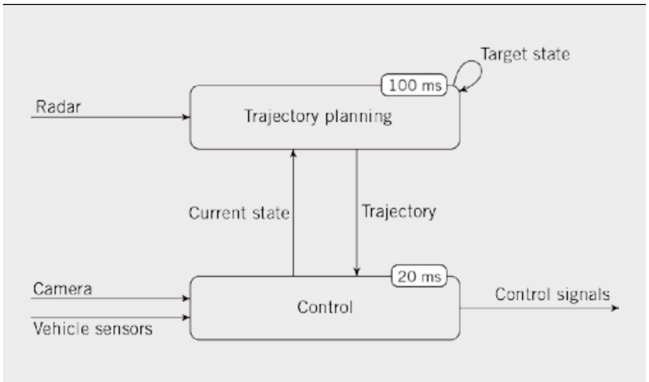

FIGURE 2 System configuration (© TU Darmstadt)

2 CONFIGURATION OF THE SYSTEM

First the sensor data is preprocessed. A road coordinate system is generated with the help of the camera data as well as the information about velocity and side slip angle. The x-direction of the coordinate system is aligned with the lane markings and the y-direction is perpendicular to them. The planning is carried out in this road coordinate system. The data of the different radar sensors is fused so that the position and dimension of the surrounding vehicles is known. Afterwards they are transformed into the road coordinate system. Surrounding vehicles occupy the complete width of the lanes they are driving on. During a lane change a surrounding vehicle can therefore occupy two lanes. By this assumption the environment can be heavily abstracted.

The system can be divided into two parts, **FIGURE 2**. The low level part controls the vehicle and is calculated every 20 ms by a real-time hardware. In the high level part new trajectories are planned every 100 ms. At the start of a lane change the current state of the vehicle is used as the initial state for the trajectory planning. In contrast, during the lane change the target state from the previous trajectory is used as the initial state. That way the trajectory outputted to the control is continuous and pursuit curves are avoided [2]. In situations where the low level control is unable to follow the trajectory the initial state has to be adapted accordingly.

The system supports the planning for three different gaps between the vehicles on the destination lane, **FIGURE 3**. For each of these gaps a trajectory is planned. Later the best trajectory is chosen. To plan one of these trajectories, the environment is reduced in such a way that only four vehicles have to be considered, **FIGURE 4**. On the start lane the vehicles in front of and behind the ego vehicle are considered. On the destination lane the considered vehicles result from the gap into which the ego vehicle should change. The vehicles are named as follows:
– FS: Front Start lane
– TS: Tail Start lane
– FD: Front Destination lane
– TD: Tail Destination lane.

3 CONCEPT

As the trajectory generation is formulated as an optimisation problem it is advantageous in terms of computation time to use as few optimisation variables as possible. To achieve this goal a special parameterisation of the trajectories is introduced. It is assumed that in front of and behind the surrounding vehicles there are safety areas which must not be used by the trajectories which are to be generated, **FIGURE 4**. The size of these safety areas depends on the velocity and takes the dimensions of the ego vehicle into account. The lateral movement onto the destination lane is only possible when and where both the start and the destination lane are free to drive on. This region will be referred to as gap from now on. This gap can be depicted in the $x(t)$ diagram drawn for an example case. For a better illustration of the gap the x-values are shown relative to an observer moving with constant velocity, **FIGURE 5**. Based on this, the lane change manoeuvre can be subdivided into three segments, **FIGURE 6**:
1. Reaching the gap on the start lane
2. Lateral movement within the gap
3. Centring on the destination lane and continuation of travelling.

Depending on the scenario, the actual trajectory consists only of a subset of these three segments. The optimisation variables are the average velocity v_1, v_2 during segments 1 and 2, as well as the speed v_e at the end of segment 2, **FIGURE 6**. From these three variables, the times t_1 and t_2 together with their corresponding positions $p_{x,1}$ and $p_{x,2}$ at the segment boundaries can be determined. The longitudinal trajectory is then described by two polynomials of degree 5 passing through the initial point P0 and the points P1($t_1|p_{x,1}$) and P2($t_2|p_{x,2}$), where the trajectory crosses the segment boundaries. For comfort reasons a continuous acceleration is required at the segment boundaries. At the end point an acceleration is demanded which is calculated by an ACC (Adaptive Cruise Control) function for the predicted traffic situation at this end point. This ensures that the switchover from the lane change assistant back to the ACC will not be noticed by the passengers after the lane change has finished. The remaining degrees of freedom are used to minimise the jerk along the trajectory.

The parameterisation of the lateral trajectory also bases on the points t_1 and t_2, that is to say the segment boundaries. The lateral

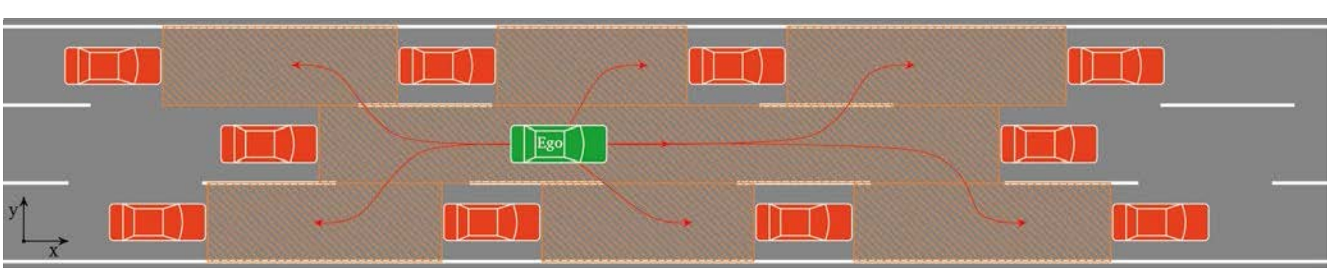

FIGURE 3 Multiple gaps per lane (© TU Darmstadt)

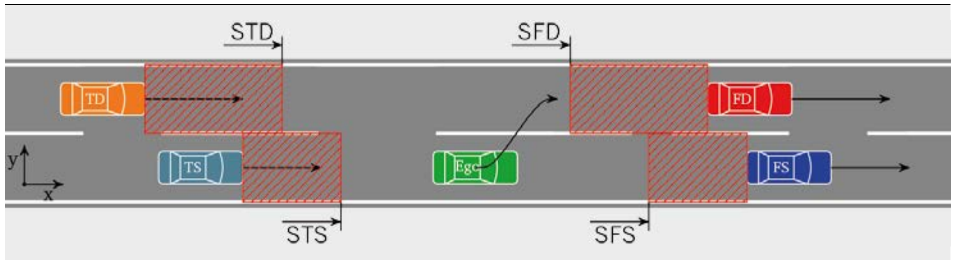

FIGURE 4 Names of the reduced surrounding vehicles to plan a trajectory for a single gap together with the safety areas of the vehicles (© TU Darmstadt)

trajectory for the segments 1 and 2 consists of polynomials of degree 5, too. For the segment 3 a curve based on a state feedback controller has proven to be favourable. The lateral trajectory is generated such that the lane change happens mainly within segment 2. The details are presented in section 4. Using this procedure the lateral trajectory is indirectly parameterised by the optimisation variables v_1 and v_2.

The complete trajectory, that is to say the longitudinal and the lateral movement, is determined by the parameters v_1, v_2 and v_e as well as the surrounding situation. This trajectory is sampled and evaluated regarding comfort, influence on the rear traffic, violation of the safety areas, and the final state. The optimal trajectory is determined by a nonlinear optimisation method. Here, the downhill simplex algorithm [6] is used.

4 DETAILED DESCRIPTION OF THE GENERATION AND EVALUATION OF TRAJECTORIES

As mentioned, the trajectory is parameterised by the average velocity during segments 1 and 2 and the velocity at the end point. Beginning at the start point P0 a line with the slope v_1 can be

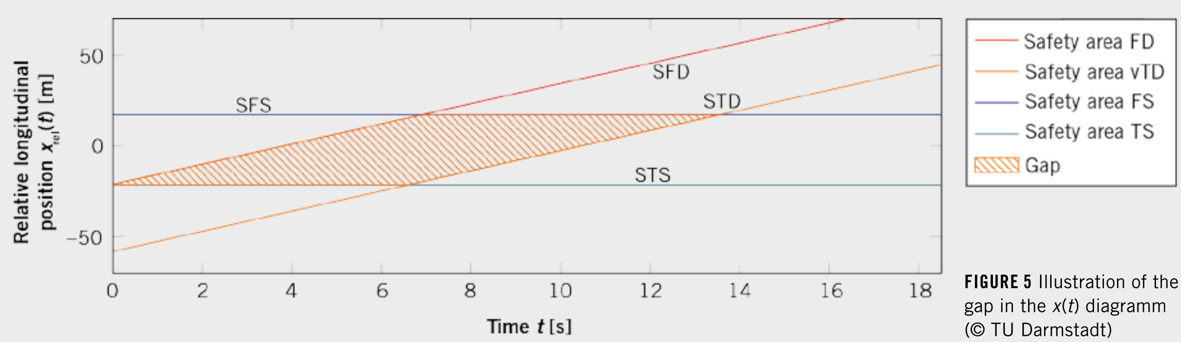

FIGURE 5 Illustration of the gap in the $x(t)$ diagramm (© TU Darmstadt)

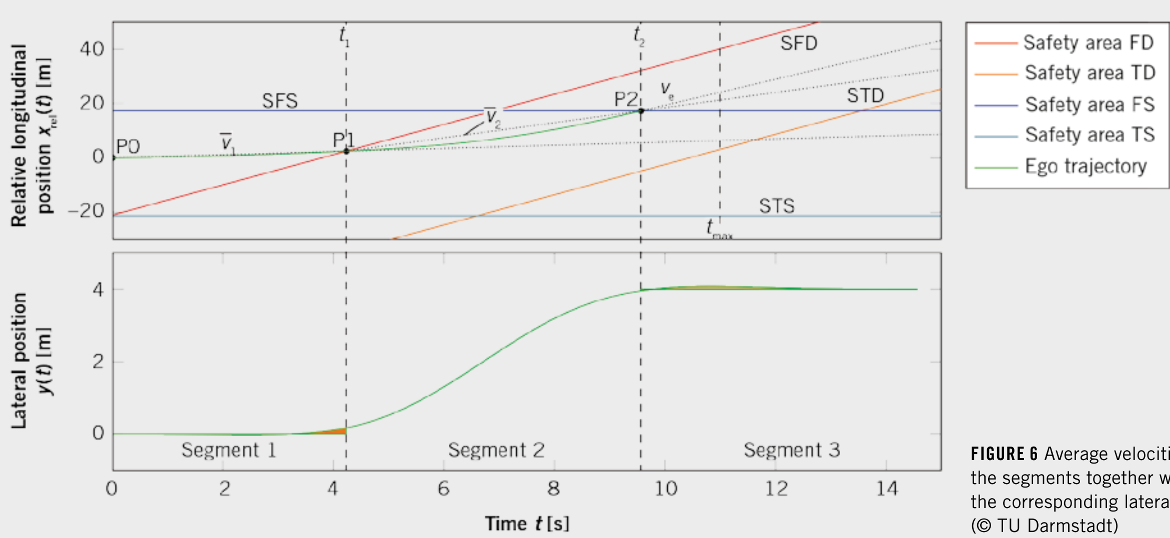

FIGURE 6 Average velocities during the segments together with the corresponding lateral motion (© TU Darmstadt)

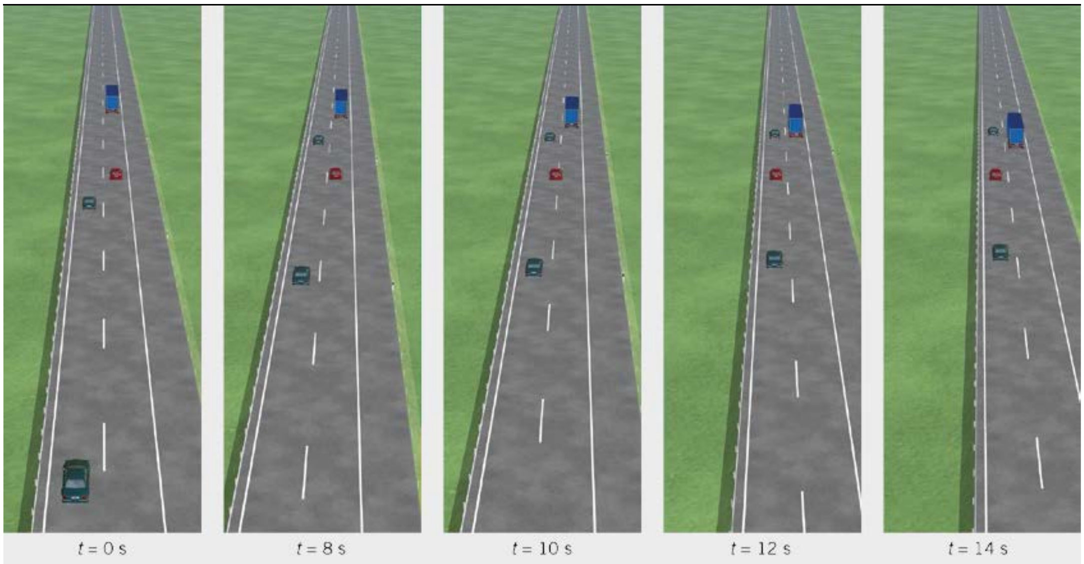

FIGURE 7 Simulation of a lane change (realistic proportions) (© TU Darmstadt)

drawn in the $x(t)$ diagram, **FIGURE 6**. The intersection of this line with the boundary of the gap defines the point $P1(t_1|p_{x,1})$. Through this point another line with the slope v_2 can be drawn. Its intersection with the leaving boundary of the gap or with a perpendicular line, which ensures a maximal duration of the manoeuvre, defines the point $P2(t_2|p_{x,2})$. The maximal duration t_{max} depends on the actual situation and is determined by the solution of an optimisation problem with linear time weighting. The velocity at the end point P2 is one of the optimisation variables. Thus, the acceleration which would be demanded by a fictitious, simple ACC based on the Intelligent Driver Model [7] can be calculated for an assumed end situation. This simplification is necessary as this acceleration has to be determined for each candidate. This acceleration value completes the boundary conditions imposed on the trajectory. Next, the coefficients of the polynomials are chosen such that these boundary conditions are met and the integral over the squared jerk is minimised. This can be done by solving a linear system of equations. Efficient algorithms are available for this task to use little computation time. The lateral trajectory also results from solving a quadratic optimisation problem. The objective function to be minimised is:

Eq. 1

$$J_{\mathrm{Lat}} = \int_{t_0}^{t_1} [w_{11}(p_y - p_{y,\mathrm{Start}})^2 + w_{12} v_y^2 + w_{13} a_y^2 + j_y^2] \mathrm{d}t + \int_{t_1}^{t_2} j_y^2 \mathrm{d}t + \int_{t_2}^{\infty} [w_{31}(p_y - p_{y,\mathrm{Ziel}})^2 + w_{32} v_y^2 + w_{33} a_y^2 + j_y^2] \mathrm{d}t$$

For the segments 1 and 3 the deviation of the lateral position p_y to the lane centres $p_{y,\mathrm{Start}}$ and $p_{y,\mathrm{Ziel}}$, the lateral velocity v_y, the lateral acceleration a_y and the jerk j_y are included in the objective function. In the segment 2 only the jerk is considered. In the segments 1 and 2 the trajectory is specified as polynomials of degree 5. In segment 3 the trajectory is calculated by a state feedback controller. This controller is designed as an optimal controller which minimised the integral from t_2 to infinity. The resulting curve is given by a matrix exponential function and is the optimal solution for the 3rd segment.

This approach has the benefit that no further coefficients of polynomials have to be determined for the last segment resulting in reducing the calculation time. Furthermore, the contribution of the segment 3 to the objective function can be calculated analytically from the final state of segment 2, that is to say it can be expressed directly by the coefficients of segment 2.

After the initial and transition conditions are incorporated six degrees of freedom remain. The corresponding coefficients are chosen such that J_{Lat} is minimised by solving a system of linear equations. The resulting trajectory is sampled and evaluated further. These additional terms are considered:

- the limited vehicle dynamics: The velocity, acceleration and jerk in both the longitudinal and lateral direction are compared to bounds. These bounds include the velocity-dependent maximum acceleration and the driver's choice of the ACC velocity.
- the influence on the rear traffic: For this the behaviour of the rear traffic on the destination lane is simulated based on the intelligent driver model. The deceleration of this vehicle is rated.
- the situation at the end of segment two: High end position and high velocity values are rewarded. Penalised is the deceleration at the end point which is necessary to avoid a collision or a violation of the chosen ACC velocity.
- the distance to the safety areas.

The individual terms are weighted and added up to give an overall objective value. Finally, the downhill simplex algorithm determines the optimal parameters v_1, v_2 and v_e which minimise the objective function.

5 RESULTS

The trajectory planning algorithm has been tested in different scenarios by means of simulation. In **FIGURE 7** an example trajectory for a change from the right to the left lane is shown. The vehicles on the left lane drive faster than the truck on the right lane. The complete lane change trajectory could be planned at $t = 0$ already, **FIGURE 8**. During the simulation run the planning adapts the trajectory to the changing surrounding situation. To get a good impression of the behaviour, also in different situations, please have a look at the online video [8]. The trajectory planning has

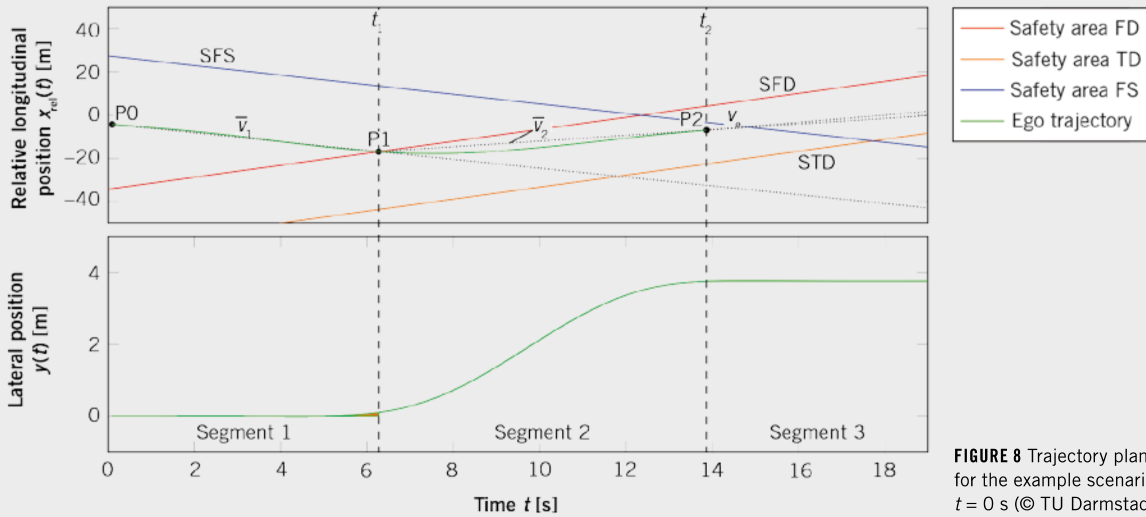

FIGURE 8 Trajectory planned for the example scenario at t = 0 s (© TU Darmstadt)

been implemented in a test vehicle. Its functionality and its real-time capability have been proven in real test drives.

6 SUMMARY

This paper describes an efficient trajectory planning algorithm for lane change manoeuvres on motorways. It is intended to cyclically plan trajectories in real-time on conventional electronic control units and with close-to-production sensors. By using the average speed as parameter for the longitudinal trajectory, it is possible to use a trajectory with multiple segments without strongly increasing the computation time. The lateral trajectory is indirectly determined by the longitudinal trajectory and the position and velocity of the surrounding vehicles. The longitudinal trajectory consists of two polynomial segments, the lateral trajectory consists of two polynomial segments and one matrix exponential function. The first segment describes the approach towards the gap. The second segment describes the lateral movement and in the last segment the remaining lateral deviation is corrected. The length of the segments is not fixed, but is derived from the average velocities. By using this method a relatively long planning horizon can be achieved. The multi-segment trajectory allows planning a trajectory whose lateral motion begins at a future point in time. Neither position nor velocity at the endpoint is predefined. They are optimised together with the average velocities and not limited to discrete values. The algorithm takes the limited dynamics of the ego vehicle and the influence onto the rear traffic into account. The trajectory planning has been tested in simulations as well as in an experimental vehicle.

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